From Alexnet to Transformers: Measuring the Non-linearity of Deep Neural Networks

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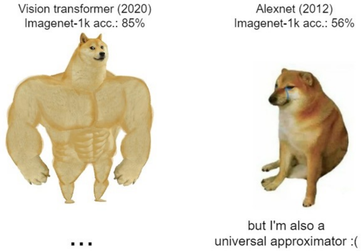
Motivation



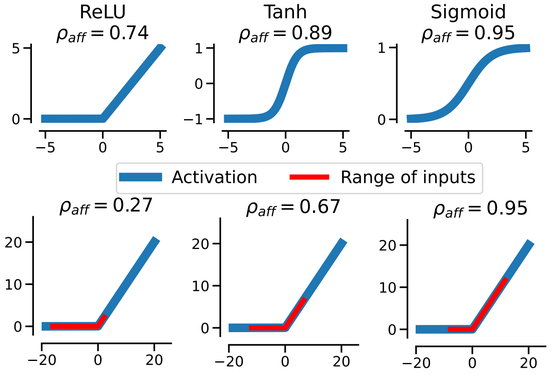
- How to **compare neural architectures** proposed over the years?

Our idea: **better understand** the intrinsic capacity of **DNNs** by **measuring their non-linearity**

Take-home message

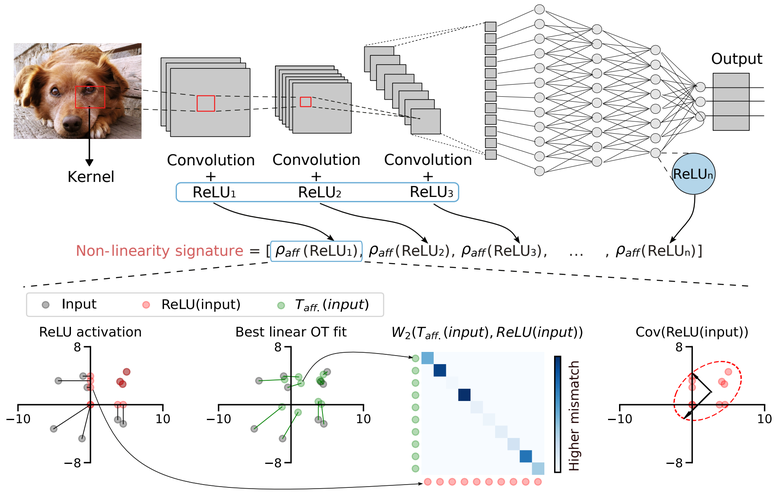
- ✓ **Theoretically sound non-linearity measure** for activations
- ✓ **Landmark DNNs** have their **distinct non-linearity signature**
- ✓ Potential applications: adversarial robustness, detection of novel disruptive models

What makes an activation more non-linear?



- **Shape** of the activation function affects its non-linearity
- **Domain of pre-activations** affects the non-linearity as well

Non-linearity signature of a DNN



Affinity score: principle tool for measuring non-linearity

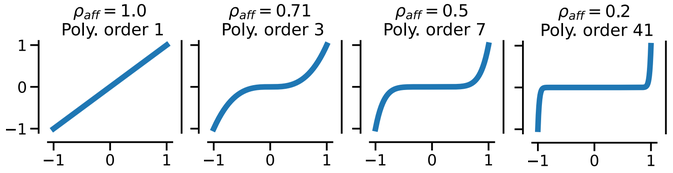
▶Let $\mathbf{X}$ be pre-activations within a DNN ▶Let $\mathbf{Y} = f(\mathbf{X})$ be output of an activation function f

Wasserstein distance — Best linear fit

$$\rho_{\text{aff}}(\mathbf{X}, \mathbf{Y}) = 1 - \frac{W_2(T_{\text{aff}}\mathbf{X}, \mathbf{Y})}{\sqrt{2\,\text{Tr}[\Sigma(\mathbf{Y})]}}$$

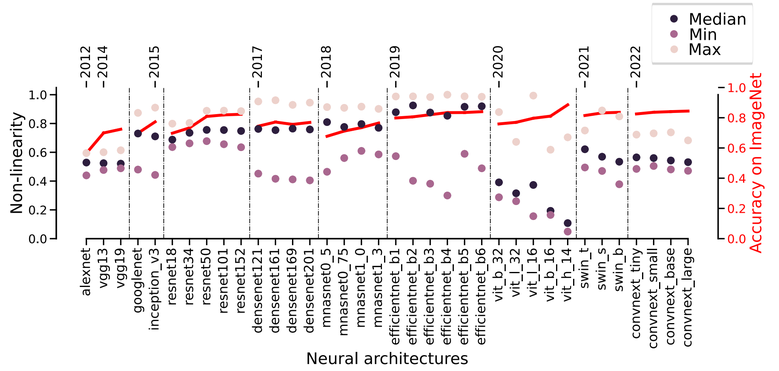
Covariance of post-activations

- ρ_{aff} = how much Y differs from being a *PSD affine transformation of* X
- T_{aff} is a globally optimal linear fit, unlike least-squares solution

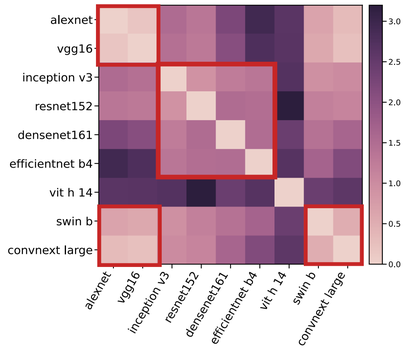


- **Lower affinity score** = transformation is **more non-linear**

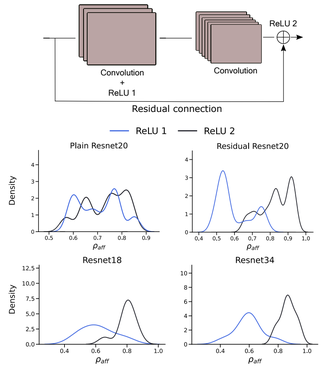
Walking through DNNs' history



Pairwise distances between DNNs



Impact of residual connections



- Before ViTs = more linear activations, more spread of ρ_{aff}.
- ViTs and after = higher non-linearity for better performance
- Residual connections = linearization of post-residual activations

Code

Paper

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